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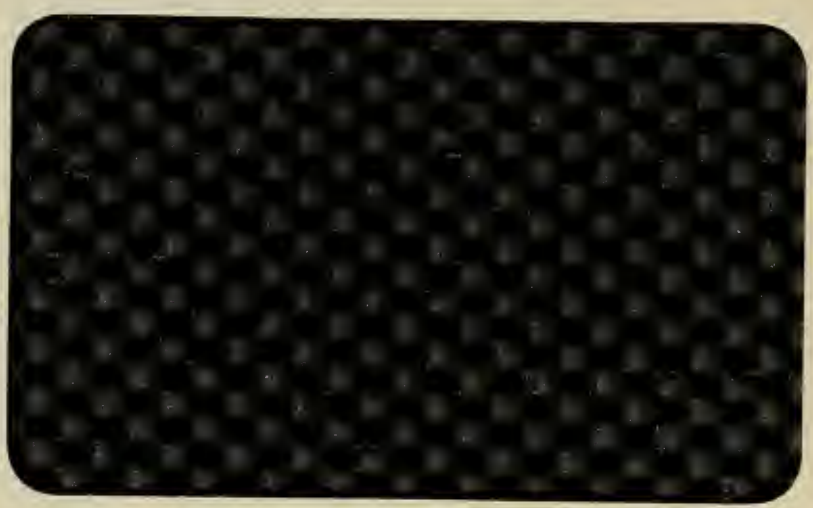
SHERWOOD PROGRESS REPORT
No. 4
(July 1959 - December 1960)

H. Grad

March 1, 1961

NEW YORK UNIVERSITY



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Introduction

This report is a summary of activities related to Project Sherwood at the Institute of Mathematicsl Sciences from July 1959 to December 1960. Substantial advances are summarized under the headings:

Propagation of Waves
Macroscopic Magneto-Fluid Dynamics
Stability
Particle Orbits
Cusped Geometries
Collisionless Shock Theory
Other Subjects

There is also a brief summary of personnel information and a bibliography of reports and papers published in the period covered. For a complete earlier bibliography see Sherwood Progress Report No. 3, NYO-2544, 1959 (hereafter referred to as PR 3).

Harold Grad
Director
Magneto-Fluid Dynamics Division

Personnel

The average number of equivalent full-time scientific staff members was approximately 19 accounted for by approximately 24 individuals who spent an appreciable amount of time on this contract. Of the 18 equivalent average man-years, some 11 are contributed by senior staff members (Ph.D.), approximately 4 by full-time research assistants, and 3 by part-time graduate students. In addition there were two active consultants and approximately 3 equivalent non-scientific persons, clerical and administrative.

Also there were three foreign scientists interested in controlled thermonuclear research visiting for a year and supported by their own organizations, and several others visiting for shorter periods.

Comparison with the statistics presented in TID-6792 shows that the Institute of Mathematical Sciences controlled thermonuclear research budget represents 1 % of the total budget and the scientific personnel on contract comprises 3 % of the total.

Propagation of Waves

A continuing and broad program is underway aimed at quantitative understanding of the reaction of a plasma to small disturbances. Two approaches are possible: the macroscopic, involving fluid-like equations, and the microscopic, arising from the Fokker-Planck or Boltzmann equations. Both lines are being pursued.

The microscopic is a more detailed, more difficult approach and involves the interactions with many aspects of plasma behavior, such as those described by the values of the plasma frequencies, gyro frequencies, collision and relaxation frequencies. Because of the complexity of this problem we must be satisfied initially with the mere evaluation of a dispersion relation, or equivelently, with the study of propagation of plane waves of infinite extent.

The relative simplicity of the macroscopic equations allows us to carry out much deeper analysis. This is essential because, as has been previously reported [PR 3], the properties of plane waves can be not only incomplete but even misleading in predicting properties of curved wave fronts and of more general disturbances.

With regard to wave fronts, the work previously reported [PR 3] was extended to describe the behavior of singular wave fronts, (D. Ludwig [26],[27]). This is not just a mathematical curiosity because, for a particular mode -- the slow wave --

this singular behavior will always appear after the wave propagates a certain distance. Together with earlier results, these new results enable one to compute the propagation of all wave fronts, including the variation in amplitude, in complete generality.

The next step in deeper understanding is the study of arbitrary disturbances as distinguished from wave fronts. Earlier work showed the existence of important interactions among the various modes which prevent their individual study in isolation [PR 3]. An important new advance was the explicit evaluation of the two-dimensional fundamental solution for Green's function, (H. Weitzner, [17], [24]). This solution exhibits the complete radiation pattern from a point source and gives the explicit interaction of all modes. Some numerical analysis will be required to discover exactly how substantial a part of a magneto-fluid dynamic disturbance propagates at speeds differing from those of the wave fronts.

Along microscopic lines, a very general formulation was made using separate Boltzmann equations (with modified collision term) for ions and electrons, (R. Liboff, [42], [67]). The initial purpose was to survey the infinity of modes, to identify those which have been found in earlier more special treatments, and to identify the new modes, previously omitted. The next result is to exhibit interconnections between previously existing fragments. For example, the macroscopic mode called transverse (the Alfven wave in an incompressible fluid) is an analytic continuation of a wave appearing in the magneto-ionic theory of modified electromagnetic radiation. Certain macroscopic modes, such as the fast wave, can

be found only in a microscopic theory such as this one, including collisions. Others make their appearances only when both ion and electron gyro frequencies are considered to be finite quantities.

The ultimate goal will be to find, in each parameter range, which are the dominant modes of propagation. It is not yet certain that they are known. Once these dominant modes have been identified, they can be analyzed more deeply, just as in the macroscopic theory.

Another approach, recently initiated, uses a simplified version of the Fokker-Planck equation instead of the Boltzmann equation with modified collision term, (A. Oppenheim). Also, certain deeper questions in wave propagation are being studied at first for the easier problem of a simple gas and a single Boltzmann equation, (J. Thurber, L. Sirovich [19]).

A special class of wave solutions in a collision-free self-consistent plasma has been found to be analogous to a problem in hydrodynamics, enabling a significant study of their properties, (C. S. Gardner and G. K. Morikawa [18]).

References quoted above: [17], [18], [19], [24], [26], [27], [42], [67].

Additional references: [7], [21], [28], [29], [51], [62], [68], [69], [71].

Macroscopic Magneto-Fluid Dynamics

The general purpose of this program is to develop quantitative mathematical tools which will eventually allow one to solve plasma flow problems with the same kind of facility an aerodynamicist has in air-flow problems. The macroscopic equations for a perfectly conducting classical fluid can be justified directly from the Boltzmann equation as applying to problems in which the relevant langth (and time) scales are long compared to all plasma properties such as mean-free-path, Larmor radius, etc., (R. Liboff, [42], [67]). However, independent of the precise domain of validity, the relative mathematical simplicity of these equations of itself makes their study very important. This situation can be likened to that of classical potential flow which is used by the aerodynamicist to establish a strong quantitative mathematical intuition about a fluid which, although virtually nonexistent, provides a foundation for the understanding of any real fluid.

The macroscopic equations bear a strong resemblance to those of ordinary fluid dynamics but also exhibit many strange and novel features. An extensive analysis has been made of those classes of conducting fluid problems which can be treated by the methods of ordinary fluid dynamics or of classical mathematical physics (H. Grad, [7]). This serves the double purpose of exhibiting a large class of problems (nonlinear and linear, transient and steady) which are accessible to computation and of separating out

those which may require significantly new methods. Some progress has been made toward understanding the latter, more unconventional problems (L. Sarason [28], also H. Grad [7]).

There is an intermediate category of problems which, although solvable by more-or-less familiar techniques, yields results which are very unusual and striking. Among these can be mentioned some of the wave propagation properties already referred to in the previous section; the transient problem of a plane wave incident upon a semi-infinite flat plate, (C. S. Gardner, [21]); certain steady flow boundary value problems, (H. Grad, [7]); and some problems in channel flow (H. Grad, [7], also C. K. Chu).

References quoted above: [7], [21], [28], [42], [67].

Additional references: [2], [14], [17], [24], [29], [35], [51], [55], [62], [68], [69].

Stability

Extensive calculations were performed to investigate the stability of a toroidal "stabilized pinch" with a sharp boundary (R. D. Richtmyer, A. Rotenberg, et al. [14], [45]). Work on this project was completed. Among other results, the computations show that the effect of bending a long thin cylinder into a torus is slightly destabilizing.

Another stability question that was investigated is the possibility that the apparent transfer from stability to instability as parameters are varied (in the problem of the stabilized pinch) may be an indication that stability is transferred from the simple equilibrium considered (cylinder) to a more complicated stable equilibrium configuration (helix), (K. O. Friedrichs, [2]). It was indeed found that a bifurcation phenomenon exists; i.e., at a transition point from stability to instability of the cylindrical equilibrium, a new equilibrium configuration branches off. However, all of the bifurcated helical equilibria found thus far (viz., those not too far from the cylindrical in shape) are unstable. Further investigation is required to discover whether greatly distorted helical equilibria exist which are stable in a geometry for which the equivalent cylindrical solution is not.

Explicit solutions were given for the equilibrium state of a low-β guiding center plasma (with non-isotropic stress tensor) in a long thin mirror geometry taking explicit account of the very non-isotropic distribution function with no particles at all

inside the loss cone (A. Oppenheim, [20]). All such equilibria are found to be unstable with regard to the $p_\perp/p_\parallel$ criterion.

References quoted above: [2], [14], [20], [45].

Additional references: [33], [36], [49], [50], [52], [56].

Particle Orbits

The guiding center motion of a particle in an electromagnetic field has been previously analyzed (M. D. Kruskal and others) as a formal expansion in the small parameter m/e. A new and greatly simplified treatment was developed using a modification of classical perturbation theory methods, (C. S. Gardner, [3]). One of the consequences is a derivation of separate Hamiltonians for the guiding center and for the circular motion about it (as formal power series). This yields very simple formal proofs of the existence of an adiabatic invariant for the circular motion (magnetic moment) and of another for the guiding center motion itself (e.g., in oscillating between mirrors); both invariants exist to arbitrarily high order under appropriate assumptions.

The significance of these formal expansions was clarified

by a proof that the guiding center expansion, although not convergent, does represent asymptotically a true solution of the exact differential equations (J. Berkowitz and C. S. Gardner [1]). As a consequence of the asymptotic rather than convergent property of this expansion, it appears that inclusion of higher order terms will give increased accuracy only in quite adiabatic problems where extreme accuracy is of interest; for a rough study of somewhat nonadiabatic orbits, higher order terms must not be taken.

The problem of nonadiabatic effects has not thus far been ameable to mathematical analysis. A computing program was initiated to study particle orbits in a special magnetic field of the cusped or picket fence configurations, (R. van Norton, [46], [66]). On the basis of these numerical experiments, it has been found possible to build a reasonable theory which is based on only a small amount of empirical (i.e., numerical) data, at least for those orbits which are not very nonadiabatic. Under appropriate circumstances, it has been verified that the degree of nonadiabaticity of an orbit is dependent almost entirely on the magnetic line to whict it belongs and depends only slightly on the value of the magnetic moment. It is also found that the changes in magnetic moment on successive passes near the low field region are statistically independent to a very high degree of approximation. Thus nonadiabatic effects in the orbits can be taken into account by introducing an equivalent diffusion coefficient whicy has an effect similar to that of coulomb scattering by other particles.

References quoted above: [1], [3], [46], [66].

Additional references: [30].

Cusped Geometries

Theoretical work in this subject was resumed in the period covered by this report, partially in response to the upsurge of experimental activity.

Considerable attention was paid to the problem of containment, in particular to the general case in which field and plasma exist together in the whole apparatus. This problem involves the study of adiabatic particles which behave just like those in a mirror machine, very nonadiabatic particles which behave somewhat like those in a sharp boundary, $\beta = 1$, cusped geometry, and the whole transition region of somewhat nonadiabatic orbits. Depending on which class of particles is dominant, the containment features can look very much like those of a mirror machine or a sharp boundary cusped geometry or (usually) like neither. This containment problem is extremely complex and has as two special limiting cases the conventional mirror machine and the sharp boundary cusped geometry.

Some early comparisons and scaling relations* were refined and improved, (H. Grad, [22]). In particular, it was shown that some parts of this analysis apply just as well to high compression mirror divices (such as the Scylla or NRL devices) as to a cusped geometry.

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* H. Grad, "The Theory of Cusped Geometries I - General Survey", NYO-7969, Institute of Mathematical Sciences (January 1959).

For purposes of simplifying computation, it was conjectured that the completely adiabatic and the strongly nonadiabatic particles are the most important (H. Grad, [22]). This was verified by numerical computation of orbits in a specific geometry, (R. van Norton, [46], [66]). Specifically, it was confirmed that, to a good approximation in the most interesting parameter range, there is a critical flux tube which separates a mirror loss region from a cusp loss region. This critical flux tube depends only on the energy of the particle. In particular, the cusp loss component of the loss rate for a finite three-dimensional device is found to be considerably less than that given by the loss formula (originally stated by Hartland Snyder) for a picket fence of infinite extent. To obtain the complete loss rate, the mirror-type losses from the adiabatic particles must be added. Thus a complete picture of the particle losses (but a rough one) has been obtained, (H. Grad, [44]). Numerical details are not quite completed.

An interesting sidelight is that the loss rate is exactly the same through the peripheral ring cusp as it is through the sum of the two axial point cusps. This means that a high compression mirror machine (in which the mean-free-path is not large) has an advantage of a factor <u>two</u> in losses over a cusped geometry; this has to be balanced against the advantage in stability for the latter, (H. Grad, [22]).

The problem of trapping of particles in a cusped geometry was studied. Some rough computations of trapping efficiency

and suggestions for experiments were made, (H. Grad, [4]). Also, the transient problem of the "filling" of a cusped device by plasma injection was studied, (J. Berkowitz, [54]).

The effect of a crossed field inside the plasma in the sharp boundary case was looked at; the loss rate is found to be reduced, (J. Killeen, [23]).

The problem of cyclotron radiation from the thin sheath in a sharp boundary plasma is being considered. The first step was to analyze the sheath itself, (H. Grad, [25]). The radiation computation is under way, (H. Burkhardt and D. Finkelstein).

References quoted above: [4], [22], [23], [25], [44], [46], [54], [66].

Additional references: [38], [57], [58], [65].

Collisionless Shock Theory

Shock waves are potentially one of the most important mechanisms for creating and irreversibly heating a plasma. The program described here is aimed at discovering whether there exist shock-like phenomena and, if so, what their properties are in a plasma in which the classical collisional mechanism is inoperative.

An analysis of a steady one-dimensional shock using an expansion in the electron-ion mass ratio was completed, (C. Morawetz, [16]). These results show an irreversible transition from a constant state in front to an oscillating state behind. This analysis applies to shocks of medium strength only and does not refer to either weak or strong shocks.

An expansion valid for weak shocks was obtained with somewhat surprising results, (C. S. Gardner and G. Morikawa, [41]). First (no surprise), a pulse was obtained rather than a shock. This solution reduces to the previously known exact solution for a zero pressure plasma. However, the width of the finite pressure pulse was found to be wider than at zero pressure in contradiction to previous approximate theories. The deviation was significant for a value of $\beta = 2\mu_o p/B^2$ larger than 1/1000. Thus, the zero pressure theory is a good approximation only for extremely low pressure (at least for weak shocks).

A refined version of a fluid theory (with non-trivial stress tensor) was carried to completion, but with inconclusive results,

(M. Rose, [15]). It is suspected that successive refinements of this type might yield an irreversible transition if the results were interpreted properly, but the situation for this type of approximation is not at all clear.

A very special problem was formulated with the feature that one could guess at plausible numerical schemes which would yield a solution to the full self-consistent problem without prior recourse to analytical simplifications of doubtful validity, (E. Keberle, [47]). The results have still to be evaluated. Some study has been made of the feasibility of direct numerical solution in more general cases, (R. D. Richtmyer), but no calculations have yet been made.

A very significant advance was made in the analysis of strong shocks, (K. W. Morton, [43]). It had been previously found that the simplest two-fluid adiabatic theory, which yielded pulse and periodic solutions for weak shocks, yielded no solution at all for strong shocks. However, in this strong shock region, it was now discovered that generalized discontinuous solutions do exist. Furthermore, numerical analysis of the transient problem of a piston moving into the plasma showed a rapid tendency to steepen and converge to these discontinuous steady solutions. Since the scale length in this problem is the familiar geometric mean of ion and electron Larmor radii, the discontinuity suggests a mechanism related to either the electron Larmor radius or the Debye radius. The discontinuity gives a definite irreversible heating, but the relative amounts going to ions and electrons

cannot be determined without studying the precise mechanism involves in the discontinuity which shows up in this crude approximation. It should be recalled that, in any shock in which the geometric mean Larmor radius alone appears, any irreversible heating must go to the ions.

A survey of the status of this subject was written, (A. A. Blank and H. Grad, [40]).

References quoted above: [15], [16], [40], [41], [43], [47].
Additional references: [34], [48], [60].

Other Subjects

A study of the thermalization of fast ions was completed, (H. Kranzer, [18]). Numerical solution of the Fokker-Planck equation showed that thermalization is a complicated process which cannot be simply described by means of a simple thermalization time, nor even by a thermalization time which depends on the initial speed of the injected ions. It was observed, for example, that an initial δ-function corresponding to a fast ion slowed and spread to a thickness considerably larger than that of the equilibrium Maxwellian and ultimately thermalized only after formation of a secondary peak at the origin.

A survey was made of the entire subject of the kinetic theory of gases, including some aspects of plasmas, (H. Grad, [5]). Among subjects included was a simplification of the bookkeeping involved in the computation of transport coefficients. It was observed that both the parallel and perpendicular resistivity of a plasma for any value of the magnetic field is easily computed from two quantitied called "first" and "second" approximations to the resistivity in the absence of a magnetic field in the Chapman-Enskog theory. Furthermore, it is easily seen without computations that the resistivity as computed by the Fokker-Planck and Boltzmann equations must be identical to cominant order.

Several studies were made of the Boltzmann equation for a simple gas of neutral particles. These results well be ultimately extended to the more involved case of a plasma. In one study, properties of the fundamental solution of the time-dependent Boltzmann equation were found, in particular near the singular region of unit Mach number, (L. Sirovich, [19]). Other studies were made of the complete spectrum of the linearized Boltzmann equation and of models with approximate collision terms in order to obtain more complete information than has been hitherto found regarding the propagation of plane waves in a simple gas. When completed, these investigations will, of course, be significant in a plasma as well.

References quoted above: [5], [8], [9].

Additional references: [6], [10], [11], [12], [13], [19], [31], [32], [39], [53], [59], [61], [63], [64], [70], [71].

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